Density of Photon states

For photon : there is no mass $\cancel{E = \frac{\hbar^2 k^2}{2m}}$

$$E = \hbar\omega$$
$$\omega = ck \quad \longrightarrow \quad \underline{E = \hbar c k}$$
$$k = \frac{2\pi}{\lambda}$$

For electron: (Spin $\pm 1/2$)

$$N(E)\,dE = \frac{\overset{\text{spin}}{2}}{(2\pi)^3}\, d^3k = \frac{2}{(2\pi)^3}\, 4\pi k^2 dk$$

for Photon : (Spin ± 1)

$$N(\omega)\,d\omega = \frac{\overset{\text{spin}}{2}}{(2\pi)^3}\, d^3k = \frac{2}{(2\pi)^3}\, 4\pi k^2 dk$$

$$N(\omega) = \frac{k^2}{\pi^2}\frac{dk}{d\omega}$$

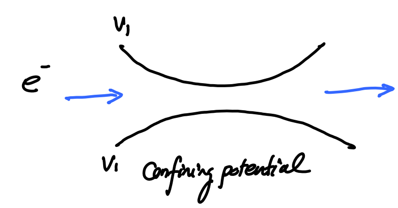
$$\omega = ck \rightarrow \frac{d\omega}{dk} = c$$

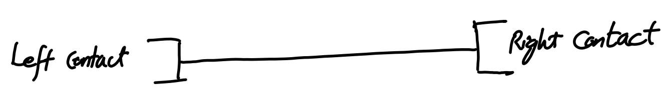
$$N(\omega) = \frac{k^2}{\pi^2 c} = \frac{\omega^2/c^2}{\pi^2 c}$$

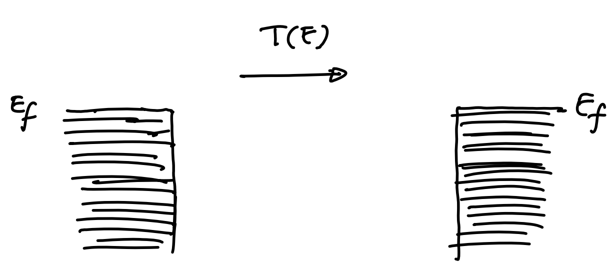
$$\boxed{N(\omega) = \frac{\omega^2}{\pi^2 c^3}}$$

Quantum Conductance

○ Calculate the current in a 1D QW.

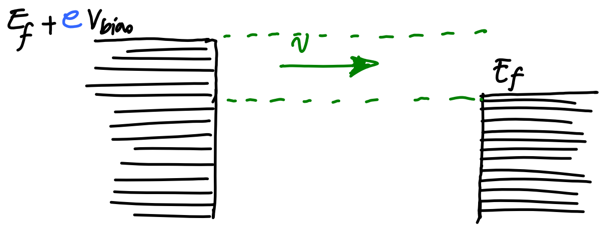






No current since there is no empty state!

Now apply a bias voltage V_{bias}:



Now electron in this range of energy can make transition between the two contacts:

$$\text{Current} \propto \begin{cases} \text{velocity } v \\ \text{Transmission probability } T \\ \text{Density of states} \end{cases}$$

$$\Rightarrow \quad I = e\int_{E_f}^{E_f+eV_{bias}} \underbrace{v(E)}_{\frac{p}{m}=\frac{\hbar k}{m}}\, T(E)\, N(E)\, dE$$

$$\overset{1D}{N(E)}dE = 2\frac{dk}{2\pi} \longrightarrow N(E) = \frac{1}{\pi}\frac{dk}{dE}$$

$$E = \frac{\hbar^2k^2}{2m} \longrightarrow \frac{dE}{dk} = \frac{\hbar^2 k}{m} \longrightarrow \frac{dk}{dE} = \frac{m}{\hbar^2 k}$$

$$\overset{1D}{N(E)} = \frac{m}{\pi\hbar^2 k}$$

$$\Rightarrow \quad I = e\int_{E_f}^{E_f+eV_{bias}} \left(\frac{\hbar \cancel{k}}{\cancel{m}}\right) T(E) \left(\frac{\cancel{m}}{\pi\hbar^2\cancel{k}}\right) dE$$

$$= \frac{e}{\pi\hbar}\int_{E_f}^{E_f+eV_{bias}} T(E)\, dE$$

If V_{bias} is small, $E_f + eV_{bias}$ is close to E_f $\Rightarrow$

$$I = \frac{e}{\pi\hbar}\int_{E_f}^{E_f+eV_{bias}} T(E_f)\, dE = \frac{eT(E_f)}{\pi\hbar}\, eV_{bias}$$

$$\Rightarrow \quad \text{Conductance } G = \frac{I}{V_{bias}} = \frac{e^2}{\pi\hbar} T(E_f)$$

$$\boxed{G = \frac{e^2}{\pi\hbar} T(E_f)}$$ Landauer Formula

In perfect transition $T(E_f) = 1 \Rightarrow$

$$G_{max} = \frac{e^2}{\pi \hbar}$$

The maximum conductance per electron per spin is:

$$\frac{e^2}{2\pi\hbar} = 25.8\ (k\Omega)^{-1} \quad \text{Universal conductance}$$

The value of quantum conductance per electron per spin is $25.8\ (k\Omega)^{-1}$.